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Specific heat jump in non-Fermi superconductors

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Abstract. We derive the jump in the specific heat at $T = T_c$ for a superconductor in a non-Fermi liquid model. We took into consideration the two possible limits in this problem: the spin-charge separation model for a Fermi liquid and the usual non-Fermi liquid model which satisfies the homogeneity relation for the spectral function $A(\Lambda\mathbf{k},\, \Lambda\omega) = \Lambda^{-1+\alpha}A(\mathbf{k},\omega)$. We also derive the order parameter behavior for these two cases in the vecinity of the critical temperature.

PACS. 74.20.Mn Nonconventional mechanisms (spin fluctuations, polarons and bipolarons, resonating valence bond model, anyon mechanism, marginal Fermi liquid, Luttinger liquid, etc.) – 74.25.-q General properties; correlations between physical properties in normal and superconducting states

1 Introduction

The unusual normal state properties in high temperature superconductors (HTSC) lead to the idea that the usual Fermi liquid theory breaks down. As a consequence there are several phenomenological models [1,2] proposed in order to explain the nonmetallic behavior of the normal state and the increases in the critical temperature. We are going to study the specific heat jump in a non-Fermi liquid system which is describe by a single particle Green's function of the form $G_0(k,\omega) = \omega_c^{-\alpha}/[\omega - u_\sigma k]^{1/2}[\omega - u_\rho k]^{1/2-\alpha}$, where ω_c is a frequency cutoff introduced to make the Green's function dimensionally correct, u_σ and u_ρ are the velocities for spin and charge density excitations and α is an non universal exponent related to the anomalous Fermi surface. This Green's function is a generalization to the higher dimensions of a fermion propagator in a 1D interacting system [3]. A similar choice of the Green's function, but with $u_\sigma = u_\rho = v_F$, was taken by Chakravarty and Anderson [4] in order to study the interlayer tunneling mechanism of the cuprate-oxides. In such a system the spectral function $A(\mathbf{k},\omega) = \mathrm{Im}G^0(\mathbf{k},\omega)$ satisfies the homogeneity relation [5] $A(\Lambda\mathbf{k},\Lambda\omega) = \Lambda^{-1+\alpha}A(\mathbf{k},\omega)$ with an exponent $\alpha > 0$, which implies the break down of the Fermi liquid theory characterized by $\alpha = 0$.

This model was used by Sudbo [6] and Muthukumar *et al.* [7] to evaluate the critical temperature in such a system. It was shown that in the spin charge separation limit ($u_\sigma \neq u_\rho, \alpha = 0$) there is an enhancement of the critical temperature compared to a Fermi liquid. In the case $\alpha > 0$ there is a critical coupling constant required for a solution of the critical temperature. For the case $\alpha \neq 0$ and $u_\sigma = u_\rho$ there have been evaluated also the Ginzburg-Landau coefficients $a(T)$ and b in order to obtain the critical field $H_{c2}(T)$ near T_c, which can be also related to the specific heat jump.

In this paper we present a calculation of the specific heat jump based on the evaluation of the thermodynamic potential near the critical temperature. The result is very similar with the one obtained in the usual BCS theory where a Fermi liquid model it works. The weak point of the model used is the fact that there are two different coupling constants in the theory, due to the generalized Green's function taken from 1D interacting system.

2 Thermodynamic potential near T_c for a non-Fermi liquid

The general formula which gives the jump in the thermodynamic potential at $T = T_c$ is given by [8]:

$$\frac{\Omega_s - \Omega_n}{V} = \int_0^{\Delta} d\Delta'(\Delta')^2 \frac{d\left(\frac{1}{g}\right)}{d\Delta'} \tag{1}$$

where Ω_s and Ω_n is the thermodynamic potential in the superconducting and normal state respectively, Δ is the superconducting gap and g is the coupling constant.

Following Sudbo [6] the general gap equation can be written as

$$\frac{1}{g} = \sum_{\mathbf{k}} \chi_0(\mathbf{k}) \tag{2}$$

where

$$\chi_0(\mathbf{k}) = -\frac{1}{\beta}\sum_{\omega_n} \frac{1}{\omega_c^{2\alpha} f_\sigma^{1/2} f_\rho^{1/2-\alpha} - \Delta_{\mathbf{k}}^2} \tag{3}$$

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with $f_\nu = (i\omega_n)^2 - (u_\nu k)^2$, $(\nu = \sigma, \rho)$, which close to T_c gives

$$\frac{1}{g} \simeq -\frac{1}{\beta\omega_c^{2\alpha}} \sum_{\mathbf{k}} \sum_{\omega_n} \frac{1}{\left[(i\omega_n)^2 - (u_\sigma k)^2\right]^{\frac{1}{2}} \left[(i\omega_n)^2 - (u_\rho k)^2\right]^{\frac{1}{2}-\alpha}} - \frac{\Delta^2}{\beta\omega_c^{4\alpha}} \sum_{\mathbf{k}} \sum_{\omega_n} \frac{1}{\left[(i\omega_n)^2 - (u_\sigma k)^2\right]\left[(i\omega_n)^2 - (u_\rho k)^2\right]^{1-2\alpha}}. \tag{4}$$

We introduce the notations $\eta = u_\sigma/u_\rho < 1$ and $\varepsilon = u_\rho k$, and if we transform the sum over k into an integral over energies we get from equation (4)

$$\frac{1}{g} \simeq -\frac{1}{\beta\omega_c^{2\alpha}} \sum_{\omega_n} \int d\varepsilon \frac{N(\varepsilon)}{\left[(i\omega_n)^2 - (\eta\varepsilon)^2\right]^{\frac{1}{2}} \left[(i\omega_n)^2 - \varepsilon^2\right]^{\frac{1}{2}-\alpha}} - \frac{\Delta^2}{\beta\omega_c^{4\alpha}} \sum_{\omega_n} \int d\varepsilon \frac{N(\varepsilon)}{\left[(i\omega_n)^2 - (\eta\varepsilon)^2\right]\left[(i\omega_n)^2 - \varepsilon^2\right]^{1-2\alpha}} \tag{5}$$

where $N(\varepsilon)$ is the energy density of states. Using now equations (1) and (5) the jump in the thermodynamic potential is

$$\frac{\Omega_s - \Omega_n}{V} \simeq -\frac{\Delta^4}{2\beta\omega_c^{4\alpha}} \sum_{\omega_n} \int d\varepsilon \frac{N(\varepsilon)}{\left[(i\omega_n)^2 - (\eta\varepsilon)^2\right]\left[(i\omega_n)^2 - \varepsilon^2\right]^{1-2\alpha}}. \tag{6}$$

There will be study two different cases. For the first one $\alpha = 0$ and $\eta \neq 1$ will correspond to a normal Fermi liquid with spin charge separation. In this limit equation (6) becomes

$$\frac{\Omega_s - \Omega_n}{V} = -\frac{\Delta^4 N(0)}{2\beta} \sum_{\omega_n} \int d\varepsilon \left[\omega_n^2 + (\eta\varepsilon)^2\right]^{-1} \left[\omega_n^2 + \varepsilon^2\right]^{-1} \tag{7}$$

where we also suppose that the density of states is constant $N(\varepsilon) = N(0) =$ constant. In order to evaluate the jump on the thermodynamic potential at $T = T_c$ in this case, first we will integrate over ε and after that we will sum over ω_n taking into account the fact that $\omega_n = (2n+1)\pi T$, is a fermionic Matsubara frequency.

Following this way we obtain

$$\frac{\Omega_s - \Omega_n}{V} = \frac{7\zeta(3)}{8} \frac{\Delta^4 N(0)}{8\pi^2} \frac{F\left(1, \frac{1}{2}; 2; 1 - \frac{1}{\eta^2}\right)}{\eta} \beta^2 \tag{8}$$

where $\zeta(x)$ is the Riemann's zeta function and $F(\alpha, \beta; \gamma; z)$ is the hypergeometric function.

The second case is the one of a non-Fermi liquid $\alpha \neq 0$ and $\eta = 1$. Using the same calculations as in the case of the Fermi liquid with spin charge separation we get the jump in the thermodynamic potential as

$$\frac{\Omega_s - \Omega_n}{V} = -\frac{2^{3-4\alpha} - 1}{2^{3-4\alpha}} \zeta(3 - 4\alpha) \times \frac{\Delta^4 N(0)}{4\omega_c^{4\alpha}} \frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{3}{2} - 2\alpha\right)}{\pi^{3-4\alpha}\Gamma(2 - 2\alpha)} \beta^{2-4\alpha}. \tag{9}$$

As we can see from equations (8, 9) an evaluation of the specific heat jump will be possible only if first we will evaluate the order parameter $\Delta(T)$ near T_c. Using the same equations we can get the usual BCS result in the limit $\eta \to 1$ and $\alpha \to 0$, respectively.

3 The order parameter $\Delta(T)$ near T_c and the specific heat jump

In both cases the order parameter $\Delta(T)$ near T_c will be calculated from equation (4) in which we replace $1/g$ from the critical temperature equation

$$\frac{1}{g} = -\frac{1}{\beta_c\omega_c^{2\alpha}} \times \sum_{\mathbf{k}} \sum_{\omega_n} \frac{1}{\left[(i\omega_n)^2 - (u_\sigma k)^2\right]^{\frac{1}{2}} \left[(i\omega_n)^2 - (u_\rho k)^2\right]^{1/2-\alpha}} \tag{10}$$

where $\beta_c = 1/T_c$. The difficulties of the problem arrive from the fact that in the case of a non-Fermi liquid the energy spectrum is not a normal one, in place of poles into the Green's function, as we have in a Fermi liquid, we have to deal with branch-cuts.

For the first case of a spin charge separated Fermi liquid we have to evaluate the sums

$$S_1 = \frac{1}{\beta} \sum_{\omega_n} \frac{1}{\left[(i\omega_n)^2 - (\eta\varepsilon)^2\right]^{1/2} \left[(i\omega_n)^2 - \varepsilon^2\right]^{1/2}} \tag{11}$$

and

$$S_2 = \frac{1}{\beta} \sum_{\omega_n} \frac{1}{\left[(i\omega_n)^2 - (\eta\varepsilon)^2\right]\left[(i\omega_n)^2 - \varepsilon^2\right]} \tag{12}$$

and we obtain for the order parameter the general equation

$$\frac{1}{\pi} \int d\varepsilon N(\varepsilon) \int_{\eta\varepsilon}^{\varepsilon} dx \frac{\tanh\frac{\beta_c x}{2} - \tanh\frac{\beta x}{2}}{\left[x^2 - (\eta x)^2\right]^{1/2} \left[\varepsilon^2 - x^2\right]^{1/2}} = -\Delta^2 \frac{N(0)}{4\pi^2} \frac{F\left(1, \frac{1}{2}; 2; 1 - \frac{1}{\eta^2}\right)}{\eta} \frac{7\zeta(3)}{8} \beta^2. \tag{13}$$

The left side of the equation (13) will be evaluated following the method proposed in reference [6]. We introduce the function

$$H(z, \eta) = \int_\eta^1 du \frac{\tanh zu}{\sqrt{(u^2 - \eta^2)(1 - u^2)}} \tag{14}$$

which can be approximated as

$$H(z, \eta) = \begin{cases} \pi z/2, & z \ll 1 \\ K(\sqrt{1 - \eta^2}), & z \gg 1 \end{cases} \tag{15}$$

where $K(x)$ is the complete elliptic integral of the first kind. Using this function the gap value near T_c is obtained in the form

$$\Delta^2(T) = \frac{4\pi\eta}{\beta^2}\frac{8}{7\zeta(3)}\frac{K\left(\sqrt{1-\eta^2}\right)}{F\left(1,\frac{1}{2};2;1-\frac{1}{\eta^2}\right)}\ln\frac{\beta}{\beta_c} \tag{16}$$

which can be approximated as

$$\Delta^2(T\to T_c) = \frac{8}{7\zeta(3)}4\pi T_c^2\frac{\eta K\left(\sqrt{1-\eta^2}\right)}{F\left(1,\frac{1}{2};2;1-\frac{1}{\eta^2}\right)}\left(1-\frac{T}{T_c}\right). \tag{17}$$

The second case, $\alpha \neq 0$ and $\eta = 1$, is more complicated and we have to evaluate the sums

$$S_1 = \frac{1}{\beta}\sum_{\omega_n}\frac{1}{\left[(i\omega_n)^2-\varepsilon^2\right]^{1-\alpha}}, \tag{18}$$

$$S_1 = \frac{1}{\beta}\sum_{\omega_n}\frac{1}{\left[(i\omega_n)^2-\varepsilon^2\right]^{2-2\alpha}}. \tag{19}$$

The general equation for the order parameter is obtained as

$$\begin{aligned}\frac{\sin\left[(1-\alpha)\pi\right]}{\pi\omega_c^{2\alpha}}&\int d\varepsilon N(\varepsilon)\int_\varepsilon^\infty dx\frac{\tanh\frac{\beta_c x}{2}-\tanh\frac{\beta x}{2}}{(x^2-\varepsilon^2)^{1-\alpha}}\\ &= \Delta^2\frac{N(0)}{2\omega_c^{4\alpha}}\frac{2^{3-4\alpha}-1}{2^{3-4\alpha}}\frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{3}{2}-2\alpha\right)}{\pi^{3-4\alpha}\Gamma(2-2\alpha)}\zeta(3-4\alpha)\beta^{2-4\alpha}.\end{aligned} \tag{20}$$

In order to evaluate the left side of equation (20) we use the series expansion $\tanh\frac{\beta x}{2} = 1-2\sum_{m=0}^{\infty}(-1)^m e^{-\beta(m+1)x}$ from which we will keep at the end only the term with $m = 0$. The order parameter in this case of a non-Fermi liquid near T_c will be

$$\begin{aligned}\Delta^2(T\to T_c) = &\frac{2\omega_c^{2\alpha}}{\beta_c^{2-2\alpha}}\frac{2^{3-4\alpha}}{2^{3-4\alpha}-1}\frac{2^{2\alpha-1}}{\zeta(3-4\alpha)}\\ &\times\frac{\pi^{3-4\alpha}\Gamma(2-2\alpha)}{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{3}{2}-2\alpha\right)}\left[\Gamma(\alpha)\right]^2\frac{\sin\pi\alpha}{\pi}\left[1-\left(\frac{T}{T_c}\right)^{2\alpha}\right].\end{aligned} \tag{21}$$

Using these results we are able to express the variation of the thermodynamic potential near the critical temperature T_c. For the spin charge separation model we get

$$\begin{aligned}\frac{\Omega_s-\Omega_n}{V} = &-2N(0)T_c^2\frac{8}{7\zeta(3)}\\ &\times\frac{\eta K^2(\sqrt{1-\eta^2})}{F\left(1,\frac{1}{2};2;1-\frac{1}{\eta^2}\right)}\left(1-\frac{T}{T_c}\right)^2\end{aligned} \tag{22}$$

and for the non-Fermi liquid model

$$\begin{aligned}\frac{\Omega_s-\Omega_n}{V} = &-N(0)T_c^2\frac{2}{\left(2^{3-4\alpha}-1\right)\zeta(3-4\alpha)}\\ &\times\frac{\pi^{3-4\alpha}\Gamma(2-2\alpha)}{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{3}{2}-2\alpha\right)}\left[\Gamma(\alpha)\right]^4\frac{\sin^2\pi\alpha}{\pi^2}\left[1-\left(\frac{T}{T_c}\right)^{2\alpha}\right]^2.\end{aligned} \tag{23}$$

The specific heat jump is related to the thermodynamic potential variation by the general relation $C_s - C_n = -T\frac{\partial^2(\Omega_s-\Omega_n)}{\partial T^2}$ which gives for the two different cases the value

$$\frac{C_s-C_n}{V} = 4N(0)T_c\frac{8}{7\zeta(3)}\frac{\eta K^2(\sqrt{1-\eta^2})}{F\left(1,\frac{1}{2};2;1-\frac{1}{\eta^2}\right)} \tag{24}$$

when $\alpha = 0$ and $\eta \neq 1$ and

$$\begin{aligned}\frac{C_s-C_n}{V} = &N(0)T_c\frac{16\alpha^2\left[\Gamma(\alpha)\right]^4}{(2^{3-4\alpha}-1)\zeta(3-4\alpha)}\\ &\times\frac{\pi^{3-4\alpha}\Gamma(2-2\alpha)}{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{3}{2}-2\alpha\right)}\frac{\sin^2\pi\alpha}{\pi^2}\end{aligned} \tag{25}$$

for the case with $\alpha \neq 0$, $\eta = 1$.

The result contained in equations (24, 25) are very similar with the result obtained in BCS calculations. The important issue of these two equations is that we can analyze the "strength" of the non-Fermi behavior by using the two parameters η and α. From the experimental point of view the fit of the data with our results can gives important information about the two parameters, or if these two parameters are important or not, which is in fact a measure of the non-Fermi character of the system.

4 Discussion

We present a derivation of the specific heat jump of a superconductor using a non-Fermi liquid model proposed to describe the strongly correlated electron system in high temperature superconductors. The same model was used by Sudbo [6] and Muthukumar *et al.* [7] in order to obtain the critical temperature corresponding to the superconductivity transition. First we derive the asymptotic formula for the order parameter $\Delta(T)$ at $T \to T_c$. We find a similar dependence with the BCS case for both cases which we studied. Using these result we find the jump in the thermodynamic potential at $T = T_c$ and we express the jump on the specific heat. We have to mention that for the case of spin charge separated Fermi liquid there is possible to reobtain the usual BCS formula for the specific heat jump by putting $\eta \to 1$ in equation (24). For the other case, the one of the non-Fermi liquid with $\alpha \neq 0$, the BCS limit is hard to be reobtained due to the fact that the limit $\alpha = 0$ is not possible to be done in equation (25). The results is difficult to be discussed in connection with experimental data because of the two coupling constants,

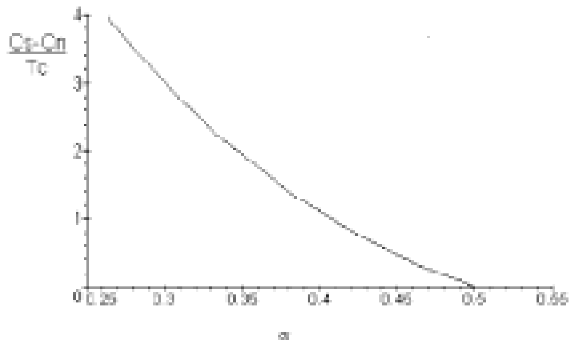

Fig. 1. Specific heat jump scaled to the critical temperature function of the coupling constant α.

α (α depends on the coupling constants of the strongly correlated electron system [10]) and λ, which characterize the model. Even in this case the results are important because we can get more information about the second coupling constant α, which is related to the unusual form of the Fermi surface. From theoretical derivation of the critical temperature [9] it was shown that α should satisfied the condition $0.3 < \alpha < 0.35$ in order to get a reasonable value for T_c. Taking some reasonable values of the specific heat jump related to the critical temperature $\Delta C/T_c$ (see Fig. 1) we get that α should satisfies the condition $0.3 < \alpha < 0.4$.

In high temperature superconductors an important parameter is the doping level. It is well known that in the underdoped, optimal doped and slightly overdoped samples the usual Fermi liquid theory breaks down, and in the strong overdoped samples the normal state properties still can be described by the usual Fermi liquid theory. So, for our results the more important case is the first one. Another problem which should be taken into consideration is the opening of the pseudo-gap in the normal state of these materials. The question which should be answered is if in a mean field treatment of the problem we deal with the pseudo-gap opening or with the real transition. However, in the case of slightly overdoped samples the transition temperature and the temperature related to the pseudo-gap opening seem to be close enough, so these problem disappears in such conditions.

Another point of view is the fact that at the transition point the phase fluctuation seem to have an important role. It was shown by Nguyen and Sudbo [11] that the role of such fluctuations can be taken into account in a 3DXY model and the results are close to the experimental data. In our calculations such an influence is difficult to be discussed due to the fact that the BCS model is a pure mean-field approximation. However, such influences on the results given by a non-Fermi liquid model can be considered in a gauge model, or a model with singular interaction, which is more difficult to be compared with our calculations.

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